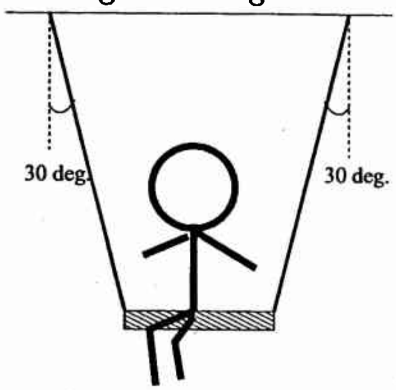
1. [30 *points*] Short Answers (6 points each)

(a) If the earth has an approximate mass of $6 \times 10^{24} kg$, what is the gravitational force that a 60kg person exerts on the earth? Please make sure you express your answer as a vector.

(b) For a particular particle,
$\vec{r}(t) = (5 + 2t - 4t^2)\hat{\imath} + (8 - 4t)\hat{\jmath}$
where position is expressed in meters, and t in seconds. What is the velocity of this particle?

(c) At $t = 1s$, how far is the particle from the origin?

(d) Consider a 40kg child sitting in a swing.



The swing is supported by two massless ropes, each 30° from the vertical (as shown). Draw a free-body diagram for the swing. You may treat the kid and the swing as a single object.

(e) What are the tensions in each of the ropes?

a) $F = ma = (60\text{ kg})(9.8\text{ m/s}^2) = \underline{588\text{ N}}$ up

b) $\vec{v} = \frac{d\vec{r}}{dt} = \underline{(2 - 8t)\hat{\imath} - 4\hat{\jmath}}$

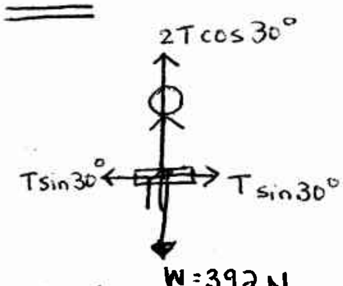
c) $\vec{r}(1) = (5 + 2(1) - 4(1)^2)\hat{\imath} + (8 - 4(1))\hat{\jmath} = 3\hat{\imath} + 4\hat{\jmath}$

$|\vec{r}| = \sqrt{3^2 + 4^2} = \underline{\underline{5\text{ m}}}$

d)

$W = mg = (40)(9.8) = 392\text{ N}$

can also be drawn as :



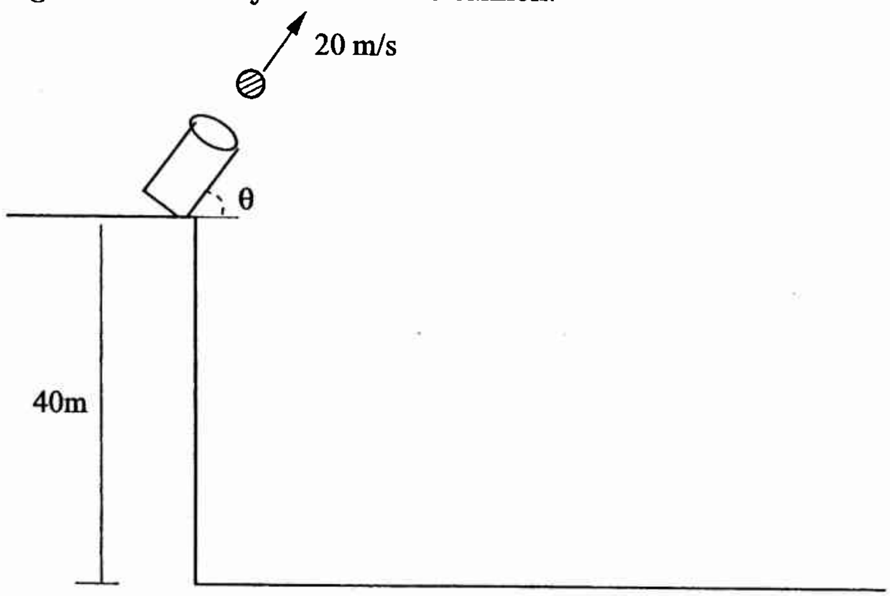
e) $\Sigma F_y = 0 = 2T\cos 30° - W$

$W = 2T\cos 30°$

$T = \frac{W}{2\cos 30°}$

$T = \frac{(40\text{ kg})(9.8\text{ m/s}^2)}{2\cos 30°} = \underline{\underline{226.3\text{ N}}}$

2. [35 *points*] You are standing at edge of a cliff, 40m above the ground, armed with a cannon (as shown). The cannon is capable of firing 15 kg cannon balls with a speed of 20m/s, regardless of how you orient the cannon.



You decide to tilt the cannon 60° above the horizontal and fire. Assume no air resistance.

(a) Expressed as a vector, what is the *velocity* of the ball the instant after it leaves the cannon?

(b) Draw a free-body diagram of the cannonball the instant after it leaves the cannon.

(c) How long after you fire does the ball reach its maximum height above the ground?

(d) How long does the ball stay aloft? (How long before it hits the ground?)

(e) How far from the cliff does the ball land?

(f) What is the velocity of the ball when it hits the ground?

(g) E.C. Instead of firing at 60°, you could have fired at any angle you like. Find the angle that produces the maximum distance from the cliff. (Hint: it's not 45°.)

a) $\vec{v} = (20 \cos 60°)\,\hat{\imath} + (20 \sin 60°)\,\hat{\jmath} = \underline{10\,\hat{\imath} + 17.32\,\hat{\jmath}}$

b)  $W = mg = (15\text{ kg})(9.8\text{ m/s}^2) = 147\text{ N}$

c) max height occurs when $v_y = 0$ $\quad v_y = v_{oy} + a_y t \quad 0 = 17.32\text{ m/s} - (9.8\text{ m/s}^2)t$

$t = \dfrac{17.32\text{ m/s}}{9.8\text{ m/s}^2} = \underline{1.77\text{ sec}}$

d) $y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$ $\quad 0 = 40 + (17.32)t + \frac{1}{2}(-9.8)t^2$

$-4.9t^2 + 17.32t + 40 = 0$

use positive root of the quadratic equation

$t = \dfrac{-17.32 \pm \sqrt{17.32^2 + 4(4.9)(40)}}{2(-4.9)} = 5.13\text{ sec}$ or $-1.\cancel{59}$ sec

$a_x = 0\ m/s^2$

e) $x = x_0 + v_{0x}t + \cancel{\frac{1}{2}a_x t^2}$

$x = 0 + (10)(5.13) = \underline{\underline{51.3\ m}}$

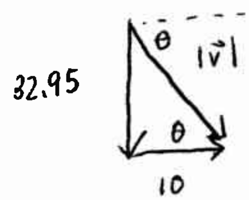
f) $v_y = v_{0y} + a_y t \qquad v_y = 17.32\ m/s - (9.8\ m/s^2)(5.13\ sec)$

$= -32.95\ m/s$

$v_x = v_{0x} + \cancel{a_x t} = 10\ m/s$

$\underline{\vec{v} = (10\ m/s)\hat{\imath} - (32.95\ m/s)\hat{\jmath}}$ or $|\vec{v}| = \sqrt{10^2 + 32.95^2} = \underline{34.43\ m/s}$

at an angle of



$\theta = \tan^{-1}\left(\frac{32.95}{10}\right)$

$= \underline{73.1° \text{ below the horizontal}}$

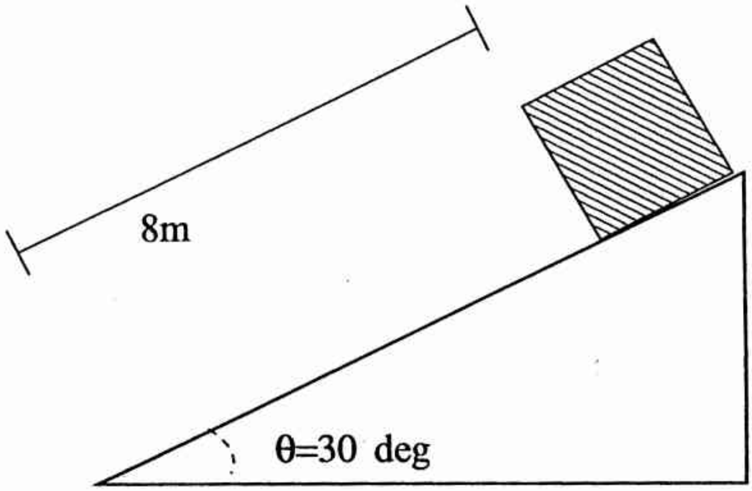
g) must have the following: $x = v_0 \cos\theta\ t$

$0 = h + v_0 \sin\theta\ t - \frac{1}{2}gt^2$

$t = \frac{+v_0 \sin\theta + \sqrt{v_0^2\sin^2\theta + 2gh}}{g}$ plug in t for x equation

must maximize x : take the derivative with respect to θ & set it equal to 0

3. [35 *points*] Consider a 20 kg block on a frictionless inclined plane with angle 30° with respect to the horizontal.



(a) Draw a free-body diagram of the block. Please break your forces into components along the plane and normal to the plane.

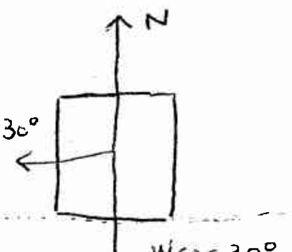
(b) What is the acceleration of the block down the plane?

(c) Suppose that the friction fairy comes along and magically grants just enough static friction to keep the block from sliding in the first place. What coefficient of static friction is required to keep the block from moving?

(d) Suppose someone gives the block a small kick and it starts moving down the plane. The coefficient of kinetic friction is 0.1. What is the *new* acceleration down the plane?

(e) Assuming that the block starts (essentially) at rest, and assuming that the plane is 8m long, how long does it take for the block to reach the bottom?

a)



$W\sin 30^\circ = (20\,kg)(9.8\,m/s^2)\sin 30^\circ = 98\,N$

plane →

$W\cos 30^\circ = (20\,kg)(9.8\,m/s^2)\cos 30^\circ = 169.74\,N$

b) $\Sigma F_{\parallel,plane} = ma_{\parallel}$ $\quad 98\,N = (20\,kg)\,a$ $\quad \underline{a = 4.9\,m/s^2}$ down the plane

c) $\Sigma F_{\parallel,plane} = 0$ $\quad -f_s + 98\,N = 0$ $\quad 98\,N = f_s = \mu_s N$ $\quad$ but $\Sigma F_{\perp plane} = 0$

$N = 169.74\,N$

$98\,N = \mu_s(169.74\,N)$ $\quad \underline{\mu_s = .58}$

d)

$\Sigma F_{\parallel, plane} = ma_{\parallel}$ $\quad f_k = \mu_k N$

$98 N - (.1)(169.74 N) = (20 kg) a_{\parallel}$ $\quad \underline{\underline{a_{\parallel} = 4.05 m/s^2}}$ down the plane

e) $\Delta x = \cancel{v_{0x} t} + \frac{1}{2} a t^2$ $\quad$ starts at rest means $v_{0x} = 0$

$8 m = \frac{1}{2}(4.05) t^2$

$t = \sqrt{\frac{2(8)}{4.05}} = \underline{\underline{1.99 sec}}$