

ELASTIC FORCES and HOOKE’S LAW

1. Objective

The objective of this lab is to show that the response of a spring when an external agent changes its equilibrium length by x can be described by Hooke’s Law, $F = -kx$. Here F is the restoring force or the force exerted by the spring on the external agent and k is a proportionality constant characteristic of the “stiffness” of the spring and is often referred to as the spring constant. We will also study the response of springs when they are connected in series and in parallel. Furthermore, it will be demonstrated that systems described by Hooke’s Law give rise to oscillatory, simple harmonic motion.

Most systems that exhibit oscillatory behavior can, to a very good approximation for small perturbations, be described by Hooke’s Law. The list of such systems is very big and varied – a pendulum, your car when you hit a bump, a marble in a bowl, atoms in a molecule or a crystal, your ear drum or any other drum, junk floating in the Schuylkill river, the quartz crystal in your watch, and electrons in the antenna of your cell phone. A mass attached to a spring is a good model system for such motion.

2. Theory

A. Single spring

From the free-body diagram in Fig. 1
 $\mathbf{F} = -\mathbf{mg} = -k\mathbf{x}$ (symbols in bold type are vectors),
 where x is the displacement from the natural equilibrium length of the vertical spring.
 Because $F = mg = kx$, k can be determined as the slope, $k = g\Delta m/\Delta x$, from the experimentally obtained m vs. x plot.

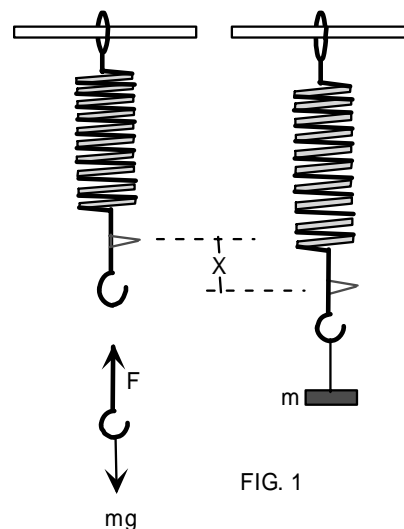


FIG. 1

B. Springs connected in parallel

Springs connected in parallel are shown in Fig.4. Using the free body diagram (FBD) in Fig. 2a

$$k_1x + k_2x = mg = k_p x$$

$$k_p = k_1 + k_2$$

where k_p is the *effective* spring constant of the two parallel springs.

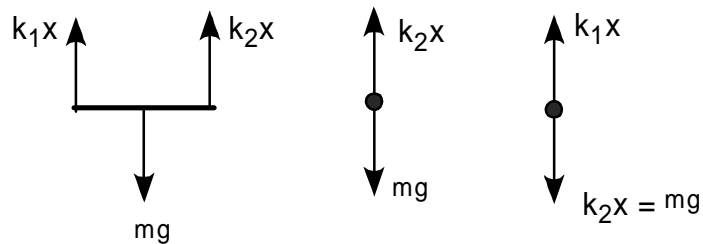


FIG. 2a

FIG. 2b

C. Springs connected in series

Springs connected in series are shown in Fig.5. Using the FBD for the spring with spring constant k_2 in Fig. 2b:

$$k_2 x_2 = mg \quad \text{or} \quad x_2 = mg/k_2$$

$$\text{Similarly, for the other spring: } k_1 x_1 = k_2 x_2 = mg \quad \text{or} \quad x_1 = mg/k_1$$

The total displacement for the series combination is $x = x_1 + x_2$. Thus, we can calculate the effective spring constant k_s for the series combination

$$k_s (x_1 + x_2) = mg$$

Substituting for x_1, x_2 from equations above the following expression results

$$k_s (mg/k_1 + mg/k_2) = mg$$

$$\boxed{1/k_s = 1/k_1 + 1/k_2}$$

2.1 Simple Harmonic Motion

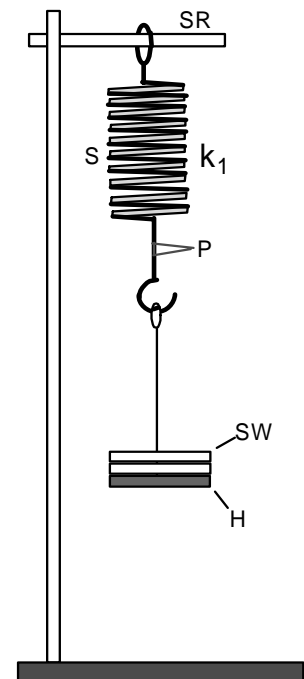
Systems whose responses can be described by $F = -kx$ undergo oscillatory sinusoidal motion, also known as simple harmonic motion (SHM). This is seen most easily by expressing $F = m d^2x/dt^2 = -kx$ or $d^2x/dt^2 + [k/m]x = 0$. Although you will study this differential equation in some detail later elsewhere, it should suffice here to note that the solution to this equation can be expressed as $x = A \sin(\omega t + \theta)$, where A is known as the amplitude of the SHM, θ is the initial phase and $\omega = [k/m]^{1/2}$ is the angular frequency. The time period of the SHM is $T = 2\pi/\omega$. [Note: To see that $x = A \sin(\omega t + \theta)$ indeed satisfies the differential equation, take the second time derivative of $x = A \sin(\omega t + \theta)$ and substitute it in the differential equation.]

3. Experimental Procedure

3.1 Apparatus

- A set of springs
- A set of slotted weights.
- A weight hanger.
- A stand with support rod.
- A meter stick, stop-watch, and a yoke.

3.2 Some general precautions.



SR - support rod
 S - spring
 P - pointer
 SW - slotted weights
 H - hanger

FIG. 3

As you conduct this experiment it is important to observe the following:

1. Never stretch the springs you are using to more than twice their relaxed length. Overstretching a spring may cause an inelastic, permanent deformation.
2. After every additional mass, allow the spring to come to rest before measuring the spring extension.

3.3 Experimental Procedure

A. Measurement of k_1 .

Suspend the spring labeled '5N' from the support rod as shown in Fig.3. Place the 100.0 cm end of the meter stick on the table (the meter readings decrease upward) and align it along the length of the spring. Read the position of the red pointer on the spring. Enter this as L_0 in data Table-I. Suspend the hanger from the free end of the spring and note the pointer position – this is entered as L_1 in Table-I. Add a 20-gm mass to the hanger and enter the pointer position as L_2 . Keep adding masses in increments of 20-gm until all entries in the Table are filled.

B. Measurement of k_2 .

Hang the spring labeled '3N' from the support rod. Repeat the steps above and fill all the entries in Table-II.

C. Measurement of k_p - the effective spring constant of springs connected in parallel.

Suspend the "5N" and the '3N' spring in parallel from the support rod as shown in Fig.4. Add various masses to the hanger as in the experiments above and enter your readings in Table-III

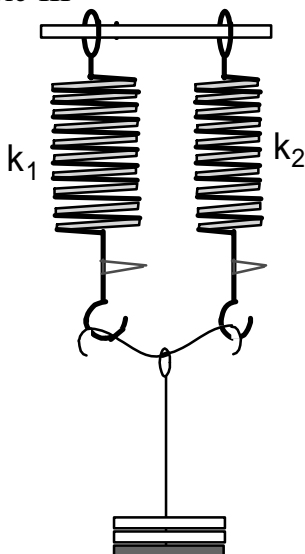


FIG. 4

D. Measurement of k_s - the effective spring constant of springs in series.

Suspend the “5N” and the ‘3N’ springs in series from the support rod as shown in Fig.5. Add various masses to the hanger as above and enter your readings in Table-VI.

E. Simple Harmonic Motion

Suspend the ‘3N’ spring from the support rod and the hanger from the free end of the spring. Add 40 g to the hanger. Holding the hanger from the bottom, pull it down to stretch the spring about 5-6 cm and let go. Using the stop watch, measure the time taken for 10 complete oscillations. Enter this reading in Table-V. Take two more readings as indicated in Table-V.

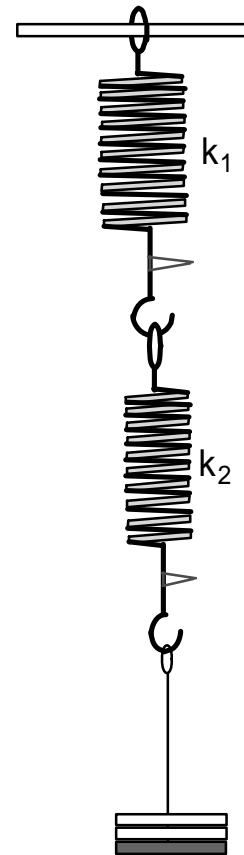


FIG. 5

Lab-02**Hooke's Law**

Name _____ Sec/Group _____ Date _____

TABLE-I
Spring constant, k_1 .

#	Spring load in grams (suspended mass, M)	Pointer Position (in cm)	Extension ΔL (in cm)
1	0	$L_0 =$	$L_0 - L_0 = 0$
2	50	$L_1 =$	$L_1 - L_0 =$
3	70	$L_2 =$	$L_2 - L_0 =$
4	90	$L_3 =$	$L_3 - L_0 =$
5	110	$L_4 =$	$L_4 - L_0 =$
6	130	$L_5 =$	$L_5 - L_0 =$
7	150	$L_6 =$	$L_6 - L_0 =$

Slope of mass M (y-axis) vs. ΔL (in cm) = _____ g/cm = _____ kg/mSpring constant, $k_1 =$ (slope in kg/m)(acceleration due to gravity in m/s^2)

= _____ N/m

Lab-02**Hooke's Law**

Name _____ Sec/Group _____ Date _____

TABLE-II
Spring constant, k_2 .

#	Spring load in grams (suspended mass, M)	Pointer Position (in cm)	Extension ΔL (in cm)
1	0	$L_0 =$	$L_0 - L_0 = 0$
2	50	$L_1 =$	$L_1 - L_0 =$
3	70	$L_2 =$	$L_2 - L_0 =$
4	90	$L_3 =$	$L_3 - L_0 =$
5	110	$L_4 =$	$L_4 - L_0 =$
6	130	$L_5 =$	$L_5 - L_0 =$
7	150	$L_6 =$	$L_6 - L_0 =$

Slope of mass M (y-axis) vs. ΔL (in cm) = _____ g/cm = _____ kg/mSpring constant, k_2 = (slope in kg/m)(acceleration due to gravity in m/s^2)

= _____ N/m

Lab-02**Hooke's Law**

Name _____ Sec/Group _____ Date _____

TABLE-III
Spring constant, k_p .
Springs connected in parallel

#	Spring load in grams (suspended mass, M)	Pointer Position (in cm)	Extension ΔL (in cm)
1	0	$L_0 =$	$L_0 - L_0 = 0$
2	50	$L_1 =$	$L_1 - L_0 =$
3	70	$L_2 =$	$L_2 - L_0 =$
4	90	$L_3 =$	$L_3 - L_0 =$
5	110	$L_4 =$	$L_4 - L_0 =$
6	130	$L_5 =$	$L_5 - L_0 =$
7	150	$L_6 =$	$L_6 - L_0 =$

Slope of mass M (y-axis) vs. ΔL (x-axis) = _____ g/cm = _____ kg/mSpring constant, k_p = (slope in kg/m)(acceleration due to gravity in $m/s^2 = 9.8 m/s^2$)

= _____ N/m

Lab-02**Hooke's Law**

Name _____ Sec/Group _____ Date _____

TABLE-IV
Spring constant, k_s .
Springs connected in series

#	Spring load in grams (suspended mass, M)	Pointer Position (in cm)	Extension ΔL (in cm)
1	0	$L_0 =$	$L_0 - L_0 = 0$
2	50	$L_1 =$	$L_1 - L_0 =$
3	70	$L_2 =$	$L_2 - L_0 =$
4	90	$L_3 =$	$L_3 - L_0 =$
5	110	$L_4 =$	$L_4 - L_0 =$
6	130	$L_5 =$	$L_5 - L_0 =$
7	150	$L_6 =$	$L_6 - L_0 =$

Slope of mass M (y-axis) vs. ΔL (in cm) = _____ g/cm = _____ kg/m

Spring constant, k_s = (slope in kg/m)(acceleration due to gravity in m/s^2)
= _____ N/m

Lab-02**Simple Harmonic Motion**

Name _____ Sec/Group _____ Date _____

TABLE-V
Oscillation Frequency

#	Suspended Mass (in g)	Time for 10 Osc. (T in sec)	Time Period $T_{ex}=T/10$ (in sec)	$T_{th} = 2\pi[m/k]^{1/2}$ (in sec)	% difference $[(T_{ex} - T_{th})/ T_{th}]*100$
1	90.0				
2	130.0				
3	170.0				

Notes: 1. $T_{th} = 2\pi[m/k]^{1/2}$, where m is the mass in kg, and k is the spring constant (in N/m) you determined experimentally

2. We have neglected the mass of the spring in all our equations. This is not completely justified. Would neglecting the spring's mass make your experimentally-determined time period $T_{ex} > T_{th}$ **or** $T_{ex} < T_{th}$?

Analysis

1. Plot a graph of mass added, M (y-axis) vs. spring extension, ΔL for the data of Table-I. Your plots should be well labeled. For a plot to be well labeled, the label along an axis should indicate the parameter, with appropriate units, being plotted. The graph should indicate the source of the data being plotted.

The plotting in this LAB will be done the same way as you did in LAB-02. Briefly, open Microsoft Excel on your computer. Enter the displacement values (ΔL 's) as a function of mass load. Enter the ΔL -values in the first column and the mass-values in the second column. Plot ΔL -values as a function of load (mass) and extract the best straight line fit. You can do this by highlighting both columns and clicking "Insert...Chart...(select XY scatter)" on the Menu Toolbar. When a graph is generated, right-click on the data points, and select "Add trendline", choosing the "Linear" option; click on Options and the Trendline window and check the option "Display equation on chart". Print out a graph for each member of your group and attach them to your Lab write-ups.

2. The spring constant $k_I = (\text{slope in kg/m}) * (\text{acceleration due to gravity, } g = 9.8\text{m/s}^2)$.
3. Repeat steps 1-3, above for Tables-II, III, and IV.
4. Determine the percentage difference between the experimentally determined spring constant k_p and the theoretical value, $k_p(th) = k_1 + k_2$

$$\Delta k_p(\%) = [(k_p - k_p(th)) / k_p(th)] * 100$$

5. Determine the percentage difference between the experimentally determined spring constant k_s and the theoretical value, $k_s(th.) = k_1 k_2 / (k_1 + k_2)$

$$\Delta k_s(\%) = [(k_s - k_s(th)) / k_s(th)] * 100$$

LAB-03 Hooke's Law

Name: _____ Sec/Group _____ Date: _____

4. Pre-Lab.

1. Are the shock absorbers in your car connected in series or in parallel?

2. Consider the function $x(t) = A\sin(\omega t + \theta)$, we discussed earlier.

[a] find the first derivative of $x(t)$.

$$dx/dt =$$

[b] find the second derivative of $x(t)$.

$$d^2x/dt^2 =$$

[c] by direct substitution show that $x(t) = A\sin(\omega t + \theta)$ satisfies the following equation, $d^2x/dt^2 + [k/m]x = 0$.

Name: _____ Sec/Group _____ Date: _____

Consider a ball suspended from the lid of a can using a string as shown in A1. Another ball is attached to one end of a spring with the other end of the spring attached to the lid of a can, see B1. Think of the can as just a mass. The reason for using a can is simply that the lower rim of the can provides a convenient visual marker for any relative vertical movement of the ball.

A. For the ball and string case

Suppose you hold the can at shoulder height and drop it. Which of the following would you expect? Make a guess and don't worry too much about being right (you are not being graded here for your answer)

[a] the string would be taut (the tension in the string, $T > 0$) and the ball would be as in A1

[b] the ball would appear as in A1, but the tension in the string would be zero.

[c] the string would go limp and the ball would move up relative to the can as in A2

B. For the ball and spring case

Suppose you hold the can at shoulder height and drop it. Which of the following would you expect? Make a guess and don't worry too much about being right.

[a] the spring and ball would maintain their position and appear as in B1. The spring would neither get compressed or stretch from its pre-release position.

[b] the ball would appear as in B2, the spring would stretch further from its pre-release position.

[c] the spring would return to its equilibrium length as in B3.

C. The reality kick's in.

Perform the experiment as described in A and B above. Write about five lines each to explain your observations. (your grade will depend on the quality of your explanation). You can use the back of this sheet for your answer.

