

Conservation Laws (Collisions)

1. Objective

The objectives of this experiment are:

- Measurement of momentum and kinetic energy in collisions.
- Experimentally test the validity of the principles of conservation of momentum and kinetic energy.
- Examine the principle of rotational invariance with respect to our conservation laws.

2. Theory

Review Chapter 8 in Serway and Jewett.

As you have learned, a closed *system* can be *characterized completely*, if one knows all the positions \vec{x} and their velocities \vec{v} of all the objects (particles) within the system, and moreover how these particles interact.

By *characterized completely*, we mean that if one knows the positions and the velocities of every object in the system and how they interact, one can (in principle) determine exactly what will happen to the system any time in the future or has happened to the system anytime in the past.

In practice of course, it is difficult to know about every ‘object’ in the system and how they interact as typically our apparatus is made of many ($\sim 10^{23}$) particles all interacting in complicated ways. Nonetheless, if we can design an experiment that is very close to this idealized system, we can ‘probe’ our experimental setup to test fundamental properties of nature such as the conservation of energy and momentum, as well as invariance principles.

All closed systems have certain properties that always hold, no matter how we

design these systems. The three properties we will investigate today are two fundamental laws:

- *The Conservation of Energy*
- *The Conservation of Linear Momentum*

And more over we will examine the

- *Rotational Invariance of the fundamental laws.*

All systems that are closed (this just means you are not exchanging mass or energy with the system) have this property. Since studying any system is quite complicated, we will study a nice simple system where we have particles (hockey pucks in our case) that do not interact unless they “bang” into each other (which will cause them to scatter about). No matter how these particles bounce, our two fundamental laws of physics will always be true. The important thing to remember here, is even though we are studying this simple system, these principles are true *in general*, for all systems!

But first lets review some physics,

Suppose I have a hockey puck with mass m and velocity \mathbf{v} . I can associate two quantities with this puck. (1) The kinetic energy of the puck:

$$E = \frac{1}{2} m \mathbf{v}^2 \quad (1.1)$$

and (2) the linear momentum of the pick:

$$\mathbf{p} = m \mathbf{v} \quad (1.2)$$

The total linear momentum of the system is just the sum of the individual momentum of each object in your system:

$$\mathbf{p}_{tot} = \sum_i \mathbf{p}_i = \sum_i m_i \mathbf{v}_i \quad (1.3)$$

And correspondingly the total kinetic energy of the system is just the sum of the Individual kinetic energies:

$$E = \sum_i \frac{1}{2} m_i \mathbf{v}_i^2 \quad (1.4)$$

To review, the energy of course is a simple number (scalar) and only has positive quantities, while the linear momentum is a vector (three numbers) and each component may have positive or negative quantities)

We are designing an experiment such that the potential energy of the system does not change (the objects are not interacting except for the collision and friction may

be neglected). Therefore the kinetic energy (eq. (1.4)) represents the total energy, which the law of conservation of energy tells us, *is conserved*.

So in our system both quantities are conserved that is the total energy and total linear momentum is a constant if that system is isolated. We must note however real experiments are of course not isolated and one might expect to observe that over a long period of time a puck moving on a table will still slowly lose energy. That is, the kinetic energy of the system will over time get converted to other forms (heat via friction) of energy and correspondingly, over a long time the same will be true for linear momentum.

The important thing to think about is if we can design an experiment that is effectively isolated, we will be able to observe our conservation laws in action. What does this mean? Well as we said earlier, one of the main ways energy and linear momentum can get transferred outside of our system is via friction. Notice how the apparatus in front of you works, the pucks are cleverly designed to sit on a cushion of air. This minimizes the friction between the puck and the table.

This gives us (to a very good approximation) an effectively isolated ‘universe’ (made of pucks), to test out our newly learned theories about how our universe actually works. Remember this is how (as far as we can tell) **everything** in the known universe works not just hockey pucks in isolation!

Lets examine the table a little more carefully to see how it works. The pucks have known masses and sit on a cushion of air (the air is provided by the tubes). The pucks are designed in such a way, that they generate marks on a piece of paper at regular intervals (the interval time is set by you). The figure below is an example of the marks generated by the pucks. Notice how after the collisions there are a series of dots representing the motion of the puck. Since you know the time interval between each successive dot and you can measure the distance between such dots, you know the velocity of each puck. (Remember this is a vector quantity!). Since you know the velocity and the mass of each puck, by eq (1.2) you know the momentum and by eq. (1.1) you know the kinetic energy both before and after the collision. If our table obeys the laws of physics, we would expect that the total momentum and the total kinetic energy of the system is conserved!

There is one subtle point here that is worth talking about. Velocity is a vector that is (in 2 dimensions)

$$\vec{v} = (v_x, v_y) \quad (1.5)$$

where v_x is the component of the velocity along the x axis and v_y is the component of the velocity along the y axis. Now you might ask your self, ‘hold on a second here, how does the universe know how I have defined my axis?’ The answer is, it

does not! Therefore, no matter **how** you pick your axis, all our conservation laws will hold (as must be the case). That is the conservation laws are ‘invariant’ to a rotation of coordinate axis. As an aside, there is a subtle, intimate relation between these ‘invariance principles’ and conservation laws. I just want to make a note of it now. We will go back to this topic at a later date!

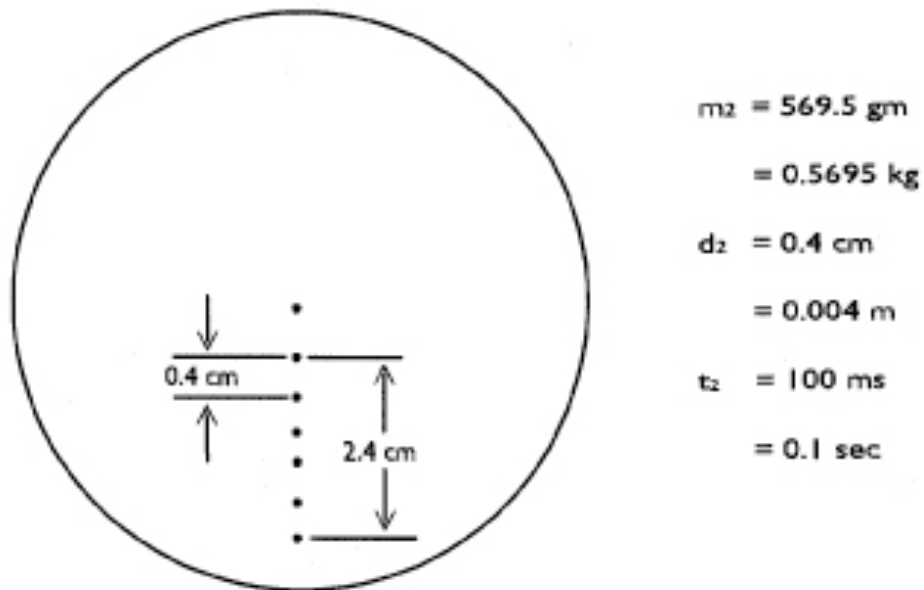


Figure 1: Sample data for used for calculating the velocity of a puck. The distance traveled over a known amount of time (the number of marks measured times 0.1 s for the example above), gives one the velocity.

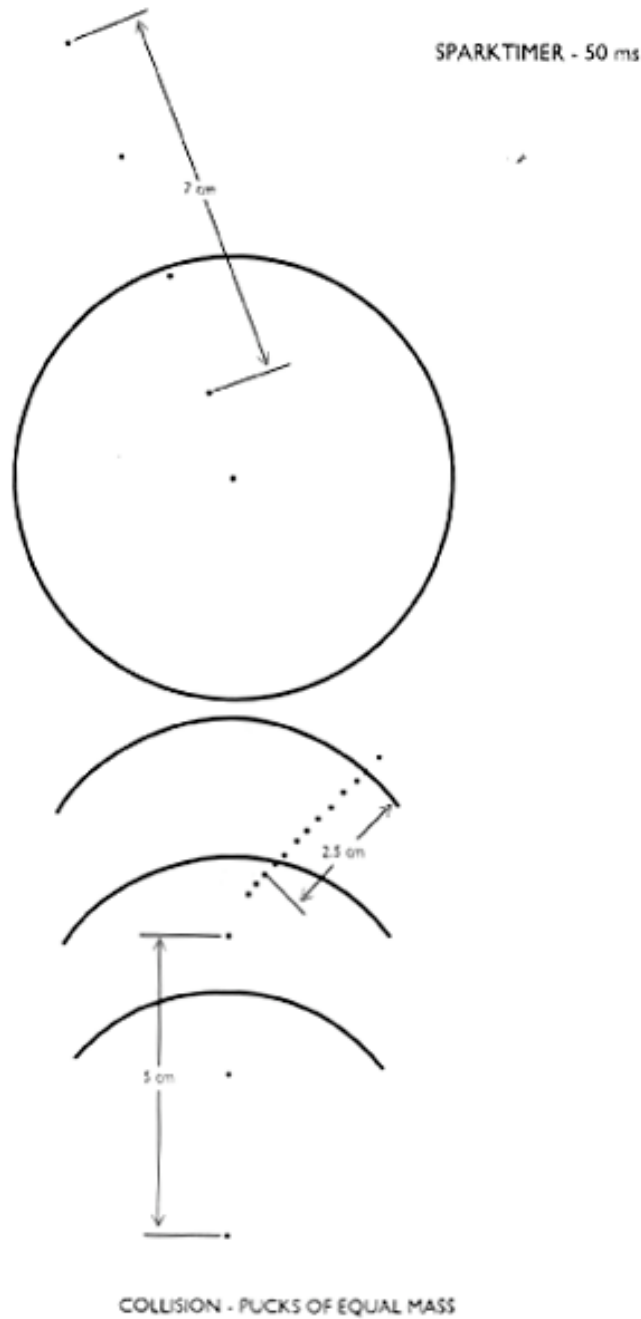


Figure 2 Example of a collision between two equal masses. As the top puck is initially still, there is only a single velocity associated with it. The bottom puck is initially moving upwards at a relatively high rate of speed, upon collision it moves towards the right at a slower rate of speed (with some of the momentum being transferred to the top puck). You should be able to determine the initial and final momentum of each puck by analyzing the above figure.

3. Procedure

1. Level the air table.
2. Set the Spark timer to the desired setting (50 ms, 20 hz) (This is the time interval between each dot placed on the paper).
3. Place a piece of recording paper on the air table and place the 0.2kg puck (P2) near the center of the table.
4. Release the 0.5kg puck (P1) to collide with the stationary puck simultaneously activating the Sparktimer.

That's it! The data you need to do this analysis is all on the paper you just generated!

Extra Credit (20 pts)

Repeat the experiment and data analysis above, only this time launch both pucks towards each other (both pucks have initial velocities so that they collide with each other near the center of the table).

You do not have to perform a rotation of coordinates for this case.

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5. Data Analysis

$m_{P1} =$ _____ $m_{P2} =$ _____

Let us first define the positive X axis as the initial direction that the first puck (the one you are throwing).

Examining the data right before the collision:

- Distance between two dots, before of the first puck before the collision?

- The 'speed' of the first puck (P1)? (Use the time interval between each mark to calculate this):

Using the "X axis" as we have defined it

- Velocity (in components) of P1? _____ **i** + _____ **j** _____

Do a similar analysis to find the same quantities for the 2nd puck (P2):

- 'Speed' of P2? _____

- 'Velocity' of P2? _____ **i** + _____ **j** _____

Now lets do the same analysis after the collision.

Since the pucks are moving in any direction now, to break the velocities into the x and y components remember to draw a right triangle and use trigonometry

- Speed of P1: _____

- Velocity of P1: _____ **i** + _____ **j** _____

- Speed of P2: _____

- Velocity of P2: _____ **i** + _____ **j** _____

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Testing the Principles of Conservation of Energy and Momentum

You are now in a position to find the momentum and the energy of the system (in either collision):

Analysis of the momenta *before* the collision ($p = mv$)

- Momentum of P1: _____ i + _____ j _____
- Momentum of P2: _____ i + _____ j _____
- Total linear momentum: _____ i + _____ j _____

Analysis of the momenta *after* the collision

- Momentum of P1: _____ i + _____ j _____
- Momentum of P2: _____ i + _____ j _____
- Total linear momentum: _____ i + _____ j _____

Analysis of the kinetic energy (in our case total energy) before the collision:

$$KE = \frac{1}{2} mv^2$$

- Kinetic energy of P1: _____
- Kinetic energy of P2: _____
- Total energy of the system before the collision: _____

Analysis of the kinetic energy (in our case total energy) after the collision:

- Kinetic energy of P1: _____
- Kinetic energy of P2: _____
- Total energy of the system after the collision: _____

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Now I want you to rotate your X axis (and Y axis) 45 degrees clockwise and do the exact same calculations:

Before the collision:

• Velocity of P1: _____ i + _____ j _____

• Velocity of P2: _____ i + _____ j _____

After the collision:

• Velocity of P1: _____ i + _____ j _____

• Velocity of P2: _____ i + _____ j _____

Analysis of the momenta *before* the collision using the 2nd coordinate system:

• Momentum of P1: _____ i + _____ j _____

• Momentum of P2: _____ i + _____ j _____

• Total linear momentum: _____ i + _____ j _____

Analysis of the momenta *after* the collision using the 2nd coordinate system:

• Momentum of P1: _____ i + _____ j _____

• Momentum of P2: _____ i + _____ j _____

• Total linear momentum: _____ i + _____ j _____

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6. Conclusions

Analyze the results you have calculated. Is the linear momentum of the system (in both coordinates) conserved after the collision? Is the total energy of the system conserved after the collision? As a note, in problems in Serway, we exploit the fact that energy and momentum are conserved to predict the velocities of the pucks after the collision! In this case, we are performing the complement experiment. We are analyzing the data representing the pre and post collision velocities to verify the conservations laws.

Why is it that I did not ask you to test the rotational invariance principle for the conservation of energy? I did not ask you not because it is not true, but because it *must* be true! Explain why this is so:

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Extra Credit (20 pts)

$m_{P1} =$ _____ $m_{P2} =$ _____

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Examining the data right before the collision:

- Distance between two dots, before of the first puck before the collision? _____

- The ‘speed’ of the first puck (P1)? (Use the time interval between each mark to calculate this): _____

Using the “X axis” as we have defined it

- Velocity (in components) of P1? _____ **i** + _____ **j** _____

Do a similar analysis to find the same quantities for the 2nd puck (P2):

- ‘Speed’ of P2? _____
- ‘Velocity’ of P2? _____ **i** + _____ **j** _____

Now lets do the same analysis after the collision.

Since the pucks are moving in any direction now, to break the velocities into the x and y components remember to draw a right triangle and use trigonometry

- Speed of P1: _____
- Velocity of P1: _____ **i** + _____ **j** _____
- Speed of P2: _____
- Velocity of P2: _____ **i** + _____ **j** _____

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You are now in a position to find the momentum and the energy of the system (in either collision):

Analysis of the momenta *before* the collision ($p = mv$)

- Momentum of P1: _____ **i** + _____ **j** _____
- Momentum of P2: _____ **i** + _____ **j** _____
- Total linear momentum: _____ **i** + _____ **j** _____

Analysis of the momenta *after* the collision

- Momentum of P1: _____ **i** + _____ **j** _____
- Momentum of P2: _____ **i** + _____ **j** _____
- Total linear momentum: _____ **i** + _____ **j** _____

Analysis of the kinetic energy (in our case total energy) before the collision:

$$KE = \frac{1}{2} mv^2$$

- Kinetic energy of P1: _____
- Kinetic energy of P2: _____
- Total energy of the system before the collision: _____

Analysis of the kinetic energy (in our case total energy) after the collision:

- Kinetic energy of P1: _____
- Kinetic energy of P2: _____
- Total energy of the system after the collision: _____