Exam 2

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Name

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TA/ Section #

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Recitation Time

May 21, 2008, 8:00am

You have 1 hour to complete the exam. Please answer all questions clearly and completely, and that you clearly indicate your final answer to each problem (put a box around the final answer, for example). Make sure that you show all of your work. If your exam sheets do not have enough room for you to write on, please ask for more paper.

You may use a calculator, and, of course, reference the formula sheet, attached. Beyond that, the exam is entirely closed book.

Formula Sheet

Physical Constants

$$\begin{aligned} G &= 6.67 \times 10^{-11} Nm^2/kg^2 \\ g &= 9.8 m/s^2 \simeq 10 m/s^2 \\ c &= 3 \times 10^8 m/s \end{aligned}$$

Some useful math relations

$$\begin{aligned} \frac{dC}{dt} &= 0 \\ \frac{d(t^n)}{dt} &= nt^{n-1} \\ \frac{d(\cos(at))}{dt} &= -a\sin(at) \\ \frac{d(\sin(at))}{dt} &= a\cos(at) \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \textit{Quadratic Formula} \end{aligned}$$

Projectile Relations

$$\begin{aligned} \Delta\vec{r} &= \vec{r}_f - \vec{r}_i \\ \vec{r} &= x\hat{i} + y\hat{j} \\ \vec{v} &= \frac{d\vec{r}}{dt} \\ \vec{a} &= \frac{d\vec{v}}{dt} \\ \vec{r}(t) &= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \\ \vec{v}(t) &= \vec{v}_i + \vec{a}t \\ v_f^2 - v_i^2 &= 2a_x\Delta x \end{aligned}$$

Circular Motion

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ a_t &= \frac{dv}{dt} \quad \textit{tangential acceleration} \end{aligned}$$

Newton's Laws

$$\begin{aligned} \vec{F} &= m\vec{a} \\ \sum\vec{F} &= 0 \quad \textit{equilibrium} \end{aligned}$$

Some specific forces

$$\begin{aligned}
F_{g,y} &= -mg \\
F_s &= -kx \\
F_{G,r} &= -\frac{GMm}{r^2} \\
F_r &= \mu F_N \\
R &= \frac{1}{2}D\rho A v^2 \ (air\ resistance) \\
V_t &= \sqrt{\frac{2mg}{DA\rho}}
\end{aligned}$$

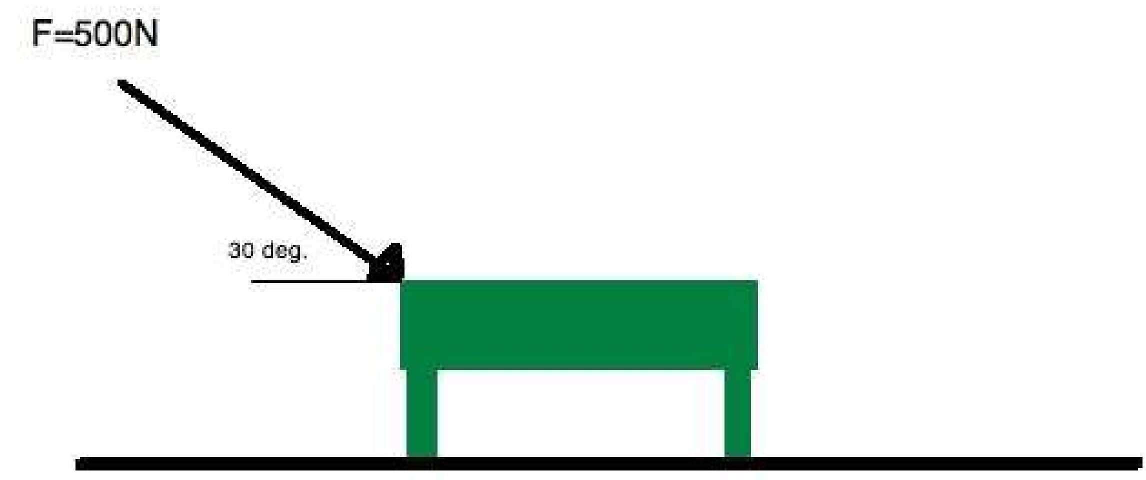
Solution of a spring

$$\begin{aligned}
x(t) &= x_0 \cos \omega t \\
\omega &= \sqrt{k/m}
\end{aligned}$$

Energy

$$\begin{aligned}
K &= \frac{1}{2}mv^2 \\
W &= \vec{F} \cdot \Delta\vec{r} \\
W &= \Delta K \\
E_{mech} &= K + U \\
-\frac{dU}{dx} &= F_x \\
E &= K + U + E_{th} \\
P &= \frac{dW}{dt} \\
&= \vec{F} \cdot \vec{v}
\end{aligned}$$

1. [35 *points*] In my spare time, I move furniture. Just yesterday I was pushing a 100kg piano (It's a little one, like the kind that Schroeder played in "Peanuts"). I pushed on it with a force of 500N at an angle of 30 degrees to the horizontal, as shown.



The coefficient of kinetic friction between the piano and floor is 0.2.

(a) What is the normal force that the floor exerts on the piano?

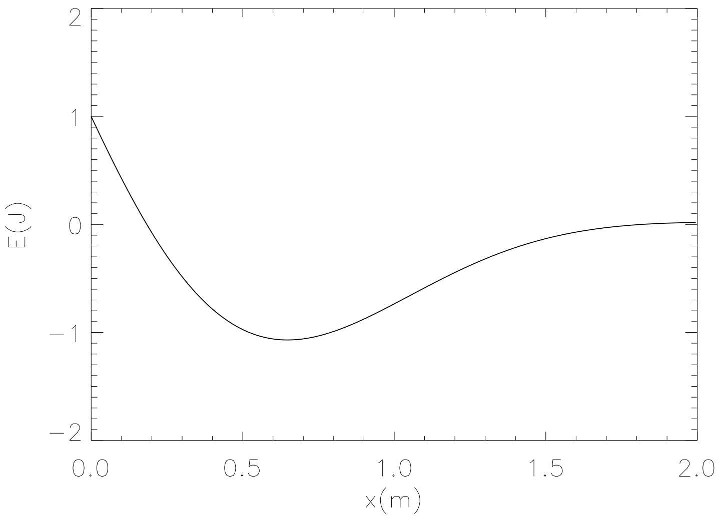
(b) Draw a free-body diagram of the piano as I push it along. Make sure you calculate all of the forces on the piano (in Newtons).

(c) I push the piano 2m before I give up and quit for the day. How much work did I do?

(d) How much work did friction do over the same distance?

(e) Assuming the piano started at rest, how fast was it going when I eventually stopped pushing it?

2. [30 *points*] Using my probe-a-fier, I am investigating a system in which a sphere can be separated by a some distance, x from a metal plate. I find that the potential energy (U) between the two as a function of distance behaves as follows.



Note that as x goes to ∞, the potential energy goes asymptotically to 0.

(a) At what (approximate) separation(s) will the system be in equilibrium? For each separation, is the equilibrium stable or unstable?

(b) Imagine that I place the sphere 1m from the plate. Is the force between them attractive, repulsive, or neutral? Please justify your answer with a sentence.

(c) Again, if I separate the spheres by 1m and let them go, what will the mechanical energy of the system be?

(d) For the same situation (i.e. that you start with an initial separation of 1m with the sphere at rest), what is the range of motion allowed to the sphere?

(e) The sphere has a mass of 0.02kg. Putting the sphere in one of the equilibria at points in part a, what the is approximate "escape speed" of the system? Hint: don't use the escape velocity for gravitational systems as found in the equation sheet. This is an entirely different system.

3. [35 *points*]

I am standing at the top of a 20m cliff, and throw a 4kg stone off horizontally with a speed of 5m/s.

(a) What is the initial kinetic energy of the stone?

(b) What is the gravitational potential energy of the stone?

(c) What is the kinetic energy of the stone when it hits the ground?

(d) **Using the projectile motion equations** (kinematic equations) what is the x-component of the velocity at the instant before the stone strikes the ground?

(e) What is the y-component of the velocity at the instant before the stone strikes the ground?

(f) At the instant before the stone hits the ground, what is the **Power** that gravity is putting into the stone?

(g) **E.C.** Knowing that the density of air is $1.29kg/m^3$, and the stone has a cross-sectional area of $0.01m^2$ and a coefficient of air resistance of 1.0, what is the force of air resistance on the stone the instant before it hits the ground? In comparison to gravity, would you expect this to be important?