PHYSICS T580: Standard Model – Final Exam

June 10, 2016, 8:00am

You have 2 hours to complete the exam. Please answer all questions clearly and completely. Make sure that you show all of your work. Only answers written in your bluebooks will be graded.

You may use a calculator, and, of course, reference the formula sheet, attached. Beyond that, the exam is entirely closed book. Unless otherwise labeled, all problem parts are given equal weighting.

1. [30 *points*] Short Answer (5 points each)

   (a) Suppose you were to come up with a new classical field theory (a GUT or a TOE) based on a Lie group. For concreteness, let's say it's SU(5). Describe in words how you would figure out how many different mediating bosons are predicted by the theory?

   E.C. Suppose the theory really *were* SU(5), based on your experience with SU(2) and SU(3), how many mediators would the theory have?

   (b) What are the implications to a classical field theory if the corresponding symmetry group of the Lagrangian is non-Abellian? What sorts of interactions arise in non-Abellian groups compared to Abellian ones?

   If you're not sure, at least explain what a non-Abellian group is.

   (c) Cosmic rays colliding with the atmosphere produce lots of muon neutrinos, ν_μ. But on the way down to a detector on earth, many of them oscillate to ν_τ. The probability of oscillation after a distance, L is:

   $$P_{\nu_\mu \to \nu_\tau} = \sin^2(2\theta_{23}) \sin^2\left(\frac{1.267(\Delta m^2_{23}/\,\mathrm{eV}^2)\ (L/\,\mathrm{km})}{(E/\,\mathrm{GeV})}\right)$$

   For a simplified model:

   - Neutrinos are produced 10km above the earth's surface (and the detector, for this purpose, may be treated as being "on the surface.")
   - The neutrinos have are monoenergetic energy of 1 GeV.
   - The mixing angle is $\sin^2(2\theta_{23}) \simeq 1$
   - Only 70% as many muon neutrinos are seen as expected.

   What is Δm^2_{23}?

   E.C. Why don't we need to consider the possible oscillations to electron neutrinos?

   (d) Without worrying overly about phase factors of i or -1, give the transformed versions of each of the following **stationary** states (through your understanding of what the bispinors mean, rather than through application of the bispinors):

   i. $\hat{\mathbf{C}} u_+$
   ii. $\hat{\mathbf{P}} u_+$
   iii. $\hat{\mathbf{C}}\hat{\mathbf{P}}\hat{\mathbf{T}} u_+$

   E.C. If your answer to part iii is anything *other* than u_+ explain in a sentence how the electroweak theory can be CPT invariant.

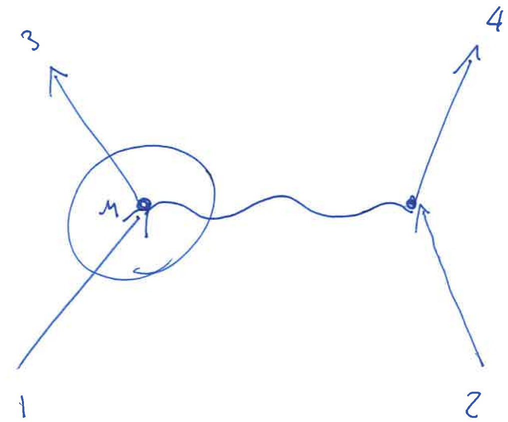
   (e) The Faraday Tensor, $F_{\mu\nu}$ is defined on the equation sheet. Please compute:

   i. F_{00}
   ii. F_{01}
   iii. F_{12}

   explicitly, in terms of components of the electric and magnetic fields.

   (f) In a sentence or two (or even a short list), explain the predictions or problems solved by electroweak unification. I'd like at least 2. Please include in at least one of your answers the role of θ_W (if not its value) and how it comes into play.

2. [20 *points*] In order to explore some properties of Dirac bispinors directly, consider a particular vertex of a Møller scattering diagram.



In this particular case, the in-going electron has a momentum of p_0 in the $+z$ direction and the outgoing electron is traveling at the same momentum an angle $\theta = \pi/2$ with respect to the z-axis.

For your convenience, the generalized (without a preferred axis) version of the u_+ bispinor is:

$$u_+(p) = \frac{m}{\sqrt{E+p^3}} \begin{pmatrix} 1 \\ 0 \\ \frac{E+p^3}{m} \\ \frac{p^1 - ip^2}{m} \end{pmatrix} \quad ;$$

(a) Assume both the ingoing and outgoing electrons are spin-up. Compute

$$[\overline{u}(3)\gamma^0 u(1)].$$

(b) How does your answer from part a change if we have a *weak* vertex:

$$\left[\overline{u}(3)\gamma^0 \frac{1-\gamma^5}{2} u(1)\right]?$$

(c) We have a device that measures the helicity of the incoming electron. Assuming the momentum is:

$$p_0 = \frac{4}{3} m_e$$

what is the probability of measuring the electron as left-handed?

(d) Let's wrap this part up by simplifying the following expression:

$$Tr(\gamma^1\gamma^2\gamma^3\gamma^1\gamma^2\gamma^3)$$

You can multiply them out explicitly if you like, but I wouldn't recommend it. If I might suggest, you might want to use the anti-commutation relations.

3. [25 *points*] In class, we spent a fair amount of time with a simplified scalar Lagrangian:

$$\mathcal{L}_{Int} = -\lambda\phi^2\eta$$

but focused on annihilation of the massive particle. Today, we're going to do particle *creation*:

$$\eta + \eta \to \phi + \phi$$

For this problem, η may taken as massless. We developed Feynman rules around this theory which can be found on the last page of the exam.

Suppose particle 1 is initially travelling at momentum p_I in the $+z$ direction, and particle 2 in the $-z$ direction.

(a) What are the momenta of the outgoing ϕ_3? Assume that particle 3 is ejected at an angle of θ with respect to the positive $z - axis$:

(b) (10 points) Draw all lowest order Feynman diagrams for this process.

(c) Calculate the total amplitude. For this part, please make sure your answer is in terms of p_I, p_F and θ.

You may find the relation:

$$\frac{1}{A - B\cos\theta} + \frac{1}{A + B\cos\theta} = \frac{2A}{A^2 - B^2\cos^2\theta}$$

helpful.

(d) *Approximate* the total cross section for the scatter in the limit of $p_I \gg m_\phi$. You may ignore any angular dependence.

4. [25 *points*] Once again, we will consider an interaction of two real-valued, scalar fields, only this time, both fields are initially massless:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \lambda\phi^2\eta - V(\eta)$$

where

$$V(\phi) = V_0 - \mu^2\eta^2 + \alpha\eta^4.$$

(a) Sketch the potential. At what value of η does this potential have a local minimum (minima)?

(b) Comparison of the normal version of a scalar field suggests that:

$$\left.\frac{d^2V}{d\eta^2}\right|_{\eta=\eta_{min}} = m^2$$

What is the mass acquired by the η field at the true vacuum?

(c) Around the minimum, we may express η as:

$$h(x) = \eta(x) - \eta_{min}$$

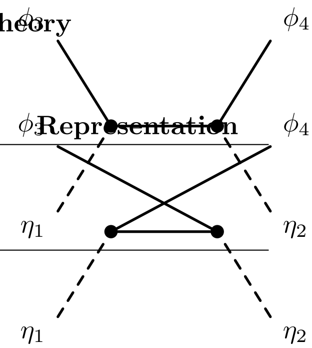
where h is a variable and η_{min} is a constant. Rewrite the Lagrangian in terms of h and ϕ only (and constants, of course).

(d) What is the mass acquired by the ϕ particle after this spontaneous symmetry breaking?

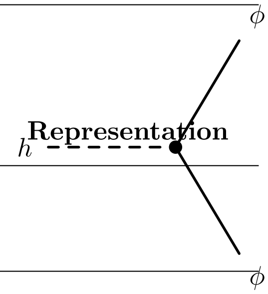
(e) Draw the Feynman vertices and give the amplitude of all $h-\phi$ interaction terms in the Lagrangian.

Feynman Rules for Scalar Theory

External Lines

Particle	Contribution	Representation
Outgoing scalar	1	
Incoming scalar	1	

Vertex Factors

Vertex	Contribution $\times \left[(2\pi)^4\delta(\Delta p)\right]$	Representation
Toy Scalar Theory	$-i\lambda$	

Propagators

Propagator	Contribution	Representation
Scalar	$\int \frac{d^4q_i}{(2\pi)^4}\frac{i}{q_i^2 - m^2}$	

yielding $-i(2\pi)^4\mathcal{A}\delta(\Delta p)$.