

Classical Mechanics I – HW 2

HW 2

Due January 31, 2019

Please answer all questions clearly and concisely. While you need not transcribe the question completely, it should be clear from your answer alone what you are talking about.

You are strongly encouraged to discuss the homework with your classmates, but you must complete the written homework by yourself, and of course, the material you submit must be your own.

Remember, show all of your work!

1. 3.3
2. 3.21
3. 3.29
4. A planet, of mass, m is orbiting a star of mass, M . At some instant in its orbit, t_0 , it is a distance, r_0 from the star, has a radial velocity, $v_{r,0}$, a tangential velocity, $v_{t,0}$.
Note: The algebra in the latter half of the problem is a bit hairy. You'll be fine, though.
 - (a) In terms of the numbers above, compute the mechanical energy, $T + U$, of the planet at t_0 .
 - (b) In terms of the numbers above, compute the angular momentum, L of the planet at t_0 .
 - (c) There are two points in the orbit known as **periastron** and **apastron** that represent the nearest and closest approach to the star. At those times, the radial component of the velocity is zero. From conservation of mechanical energy and angular momentum, compute the distance from the star at apastron and periastron in terms of M , r_0 , $v_{r,0}$, and $v_{t,0}$.
 - (d) What is the kinetic energy of the star at periastron and apastron? You may express your answer in terms of r_{ast} and r_{peri} .
5. 4.4
6. 4.8
7. 4.13
8. Let's get a little practice taking divergence, gradient, and curl, since we're going to need to do these things later. Let's define a couple of scalar functions (one in Cartesian coordinates, and one in spherical) and a couple of vectors:

$$\begin{aligned}f(x, y, z) &= x^2 + y^2 \\g(r, \theta, \phi) &= r \cos \theta \\ \vec{u} &= y\hat{i} - x\hat{j} \\ \vec{v} &= y\hat{i} + x\hat{j}\end{aligned}$$

Please calculate:

- (a) $\vec{u} \times \vec{v}$
- (b) $\nabla \times \vec{u}$
- (c) $\nabla \times \vec{v}$
- (d) $\nabla \cdot \vec{u}$
- (e) $\nabla \cdot \vec{v}$

(f) ∇f

(g) ∇g

(h) Could either of \vec{u} or \vec{v} describe a conservative force (generated by $-\nabla U$)?

9. 4.30

10. 4.53