1. Please consider the following universes:
   1) A flat $\Omega_{\gamma} = 1$ universe
   2) An empty universe
   3) A flat $\Omega_M = 0.32, \Omega_\Lambda = 0.68$ Universe.

   (a) In units of the inverse Hubble constant, how old are each of these universes?
   (b) For each of these universes, what is the Horizon distance right now?
   (c) For the concordance model (#3), create a plot of $d_{\text{hor}}$ versus $a$ on a log-log plot. Be sure to extend to $a = 10$. Do you notice anything interesting?

2. Now consider a universe with $\Omega_{\text{rel}} = 8.4 \times 10^{-5}$, and other values given by the concordance model.

   (a) At very early times ($a << 1$, such that we can ignore anything other than radiation), what is the relationship between horizon scale and $a$?
   (b) For the model just discussed, what was the comoving horizon size at $z = 1100$? This period was known as “recombination.”
   (c) What angle does that region subtend on the sky? (e.g. a patch $d_{\text{hor}}$ across at a redshift of 1100).
   (d) What angle would the patch subtend on the sky in a universe with a concordance value of $\Omega_M$, but with $\Omega_\Lambda = 0$? (In case you need a reminder, ask yourself whether such a universe is flat or not).

3. You have a set of standard candles of fixed absolute magnitude. As a “reminder,” the distance modulus relates the absolute magnitude, $M$, to the apparent magnitude, $m$, via:

   $$m - M = 5 \log_{10} d_L(\text{pc}) - 5$$

   The distance modulus is the difference between the two, and is in principle a measurable quantity. You don’t need to know how magnitudes work to compute this quantity. For the domain of $z = 0.01$ to $z = 1$, please compute the following on a log-log plot (you may overlay the first three):

   (a) The distance modulus vs redshift for an Einstein-deSitter Universe.
   (b) The distance modulus vs. redshift for a deSitter Universe.
   (c) The distance modulus vs. redshift for an Empty Universe.
   (d) On a log-linear plot, subtract out the distance modulus for a an Einstein-deSitter universe, and plot the residual. (e.g. modulus(deSitter)-modulus(Einstein-deSitter)).
4. Consider an object observed at $z = 1$ in an Einstein-deSitter Universe.

(a) What is the expansion factor at the redshift of the source?
(b) What is the comoving distance to the source?
(c) What is the lookback time?
(d) What is the Angular diameter distance?
(e) What is the Luminosity distance?

5. According to the Planck 5-year results, $\Omega_K \simeq -0.0003 \pm 0.0065$. If, in fact, the universe has a value of $\Omega_K = -0.007$ (the lower limit), what is the radius of curvature, and what is the geometry of the universe?

6. Ryden 6.9