PHYSICS 432/532: Cosmology –
Midterm Formula Sheet

Not every equation here will actually be needed on
the exam, and some may be needed more than once.
Process of elimination is not a terribly good strategy.
Also, you will be required to use equations which can
be non-trivially derived from those below. Don’t be
surprised. You are expected to know parameters de-
scribing the real universe.

Physical Constants:

- \( c = 3 \times 10^8 \text{ m/s} = 3 \times 10^5 \text{ km/s} \)
- \( G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \)
- \( h = 1.05 \times 10^{-34} \text{ Js} \)
- \( k_B = 8.62 \times 10^{-5} \text{ ev/K} \)

Units:

- \( 1 \text{ pc} = 3.086 \times 10^{16} \text{ m} = 3.26 \text{ ly} \)
- \( 1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m} \)
- \( M_\odot = 2 \times 10^{30} \text{ kg} \)
- \( t_H = \frac{1}{H_0} \simeq 9.78h^{-1} \text{ Gyr} \)
- \( d_H = \frac{c}{H_0} = 3h^{-1} \text{ Gpc} \)

Definition of the Hubble Constant:

\( H_0 \equiv \left. \frac{\dot{a}}{a} \right|_{\text{today}} = 100 \text{ km/s/Mpc} \)

Hubble’s Law (for very nearby galaxies):

\( d \simeq \frac{cz}{H_0} \)

Critical Density:

\( \rho_c = \frac{3H_0^2}{8\pi G} \simeq 1.88h^2 \times 10^{-26} \frac{\text{kg}}{\text{m}^3} \)

Density parameter:

\( \Omega_X = \frac{\rho_X(t_0)}{\rho_c} \)

Cosmological redshift:

\( a_{emitted} = \frac{1}{1+z} \)

Equation of state:

\( w \equiv \frac{P}{\rho c^2} \)

Evolution of density:

\( \rho(a) = \rho_0 a^{-3(1+w)} \)

Evolution of CMB temperature:

\( T_\gamma = \frac{T_0}{a} \)

Friedmann Equation:

\( H^2 = H_0^2 \left( \frac{\Omega_M}{a^3} + \frac{\Omega_\gamma}{a^4} + \frac{\Omega_\Lambda}{a^2} \right) \)

Acceleration Equation:

\( \ddot{a} = -\frac{4\pi G(\rho + 3P)a}{3} \)

\( q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda \)

\( S_k(\chi) = \begin{cases} R_0 \sin(\chi/R_0) & ; k = +1 \\ \chi & ; k = 0 \\ R_0 \sinh(\chi/R_0) & ; k = -1 \end{cases} \)

\( R_0 = \frac{c}{H_0 \sqrt{|\Omega_K|}} \)

Comoving Distance:

\( \Delta r = a \Delta \chi \)

\( \chi = c \int_{a_1}^{a} \frac{da}{a^2 H(a)} \)

Lookback time:

\( t_{\text{lookback}} = \int_{a_1}^{a} \frac{da}{a H(a)} \)

Angular Diameter Distance:

\( D_A \equiv \frac{S_k(\chi)}{1+z} = \frac{l_{\text{phys}}}{\theta} \)

Luminosity Distance:

\( D_L = (1+z)S_k(\chi) = \left( \frac{L}{4\pi f} \right)^{1/2} \)

Circular Velocity:

\( v^2 = \frac{GM(R)}{R} \)

Einstein Radius:

\( \theta_E = \sqrt{\frac{4GM}{c^2 D}} \)