PHYSICS 631: General Relativity

Homework # 2

Due April 25, 2017

Please show all of your work. By all means, if you have any questions, please come see me.

1. A particle in Minkowski space travels along a trajectory:

$$\begin{aligned} x(\tau) &= \alpha\tau^2 \\ y(\tau) &= \tau \\ z(\tau) &= 0 \end{aligned}$$

   (a) What are the *spacelike* components of the 4-velocity, U^i? In this part, do not give me U^0.

   (b) Using the relation $U \cdot U = -1$, compute U^0.

   (c) What is the 3-velocity of the particle as a function of τ?

2. 3.24 part a. (You will need to read up on symmetric tensor notation)

3. 3.30

4. 4.17

5. 4.21

6. Consider a stationary, ideal fluid of the form:

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

   For the moment, you should assume that the stress-energy tensor is constant in time and uniform throughout space.

   (a) Compute the stress-energy tensor, $T^{\overline{\mu\nu}}$ in a frame moving at speed, v with respect to the rest frame along the x-axis.

   (b) Suppose the pressure is a fixed ratio of the density. Compute the stress-energy tensor in the moving frame for:

      i. $P = 0$ (dust)

      ii. $P = \frac{1}{3}\rho$ (radiation)

      iii. $P = -\rho$ (cosmological constant)

   (c) Consider the fluid in the rest frame (and in generality, with P not as a function of ρ). Assume that the density and pressure of the fluid is instantaneously constant in time, but variable in space. Expand the continuity equation(s):

$$T^{\mu\nu}_{,\mu} = 0$$

   explicitly. You should use symbols like $\dot{\rho}$, ∇ and the like.

   (d) Do the same thing for the stress-energy tensor for the dust in the moving frame.