Homework # 2
Due April 25, 2017

Please show all of your work. By all means, if you have any questions, please come see me.

1. A particle in Minkowski space travels along a trajectory:
   \[ x(\tau) = \alpha \tau^2 \]
   \[ y(\tau) = \tau \]
   \[ z(\tau) = 0 \]
   (a) What are the spacelike components of the 4-velocity, \( U^i \)? In this part, do not give me \( U^0 \).
   (b) Using the relation \( U \cdot U = -1 \), compute \( U^0 \).
   (c) What is the 3-velocity of the particle as a function of \( \tau \)?

2. 3.24 part a. (You will need to read up on symmetric tensor notation)

3. 3.30

4. 4.17

5. 4.21

6. Consider a stationary, ideal fluid of the form:
   \[
   T^{\mu\nu} = \begin{pmatrix}
   \rho & 0 & 0 & 0 \\
   0 & P & 0 & 0 \\
   0 & 0 & P & 0 \\
   0 & 0 & 0 & P
   \end{pmatrix}
   \]

   For the moment, you should assume that the stress-energy tensor is constant in time and uniform throughout space.
   (a) Compute the stress-energy tensor, \( T^{\mu\nu} \) in a frame moving at speed, \( v \) with respect to the rest frame along the x-axis.
   (b) Suppose the pressure is a fixed ratio of the density. Compute the stress-energy tensor in the moving frame for:
      i. \( P = 0 \) (dust)
      ii. \( P = \frac{1}{3} \rho \) (radiation)
      iii. \( P = -\rho \) (cosmological constant)
   (c) Consider the fluid in the rest frame (and in generality, with \( P \) not as a function of \( \rho \)). Assume that the density and pressure of the fluid is instantaneously constant in time, but variable in space. Expand the continuity equation(s):
      \[ T^{\mu\nu}_{,\mu} = 0 \]
      explicitly. You should use symbols like \( \dot{\rho} \), \( \nabla \) and the like.
   (d) Do the same thing for the stress-energy tensor for the dust in the moving frame.