Homework # 5
Due May 30, 2017
Please show all of your work. By all means, if you have any questions, please come see me.

1. Consider a 1+1 dimensional space, \((t,x)\), with the metric:

\[
g_{\mu \nu} = \begin{pmatrix} -e^{kx} & 0 \\ 0 & 1 \end{pmatrix}
\]

where \(k\) is a dimensional constant. You may even want to consider what the units of \(k\) must be.

Near the origin \((x=0)\), this clearly produces the Minkowski metric.

(a) This metric has a stress-energy source which is (potentially) non-zero. Knowing nothing else, what is the scaling of the density, \(\rho\) in terms of \(k\)? (This is a dimensional analysis question, just in case you missed it).

(b) Compute all non-zero Christoffel symbols.

(c) A massive particle is instantaneously at rest at \(x = 0\). What is the instantaneous acceleration on the particle?

(d) Compute the non-zero components of the Riemann tensor. You need not compute terms that are related to one another via the various symmetry relations. Please compute in the form \(R^\alpha_{\beta \mu \nu}\).

(e) What are the non-zero terms in the Ricci Tensor and Ricci Scalar?

(f) What is the Einstein Tensor?

2. In the generalized linear metric that we found in class:

\[
g_{\mu \nu} = \begin{pmatrix} -1 - 2\psi & 0 & 0 & 0 \\ 0 & 1 - 2\phi & 0 & 0 \\ 0 & 0 & 1 - 2\phi & 0 \\ 0 & 0 & 0 & 1 - 2\phi \end{pmatrix}
\]

where, for a non-relativistically moving source:

\[
\nabla^2 \psi = 4\pi(\rho + 3P) \quad ; \quad \nabla^2 \phi = 4\pi\rho
\]

Suppose you were in the interior of a spherically symmetric distribution (a cloud) with constant density (in time and, within the cloud, in space), and fixed equation of state, \(w = -1/3\).

(a) What is the acceleration on a test particle placed a distance, \(r\) from the center of the cloud. Would it fall inward or outward?

(b) What is the acceleration on a photon traveling perpendicular to the cloud (at the speed of light, naturally), also a distance, \(r\) from the center. Would it be lensed inward or outward?

3. 8.17

4. 8.18

5. 10.9 In addition, compute the circumference of c) The proper circumference of a circle at radius, \(\tau\), and d) The proper distance in traveling from \(\tau\) to \(\tau + d\tau\).

6. Take a close look at 8.11, but no need to do it.