Classical Mechanics I – HW 4

HW 4
Due February 28, 2017 – Note longer deadline because of the midterm

Please answer all questions clearly and concisely. While you need not transcribe the question completely, it should be clear from your answer alone what you are talking about.

You are strongly encouraged to discuss the homework with your classmates, but you must complete the written homework by yourself, and of course, the material you submit must be your own.

Remember, show all of your work!

1. 7.3
2. 7.8
3. Consider a small marble of mass, \( m \), rolling around in a parabolic bowl with a curvature given by:

\[ y = \frac{r^2}{R} \]

where \( R \) is a fixed scale, and \( r \) is the radial distance from the central axis, as shown here:

(a) Write down down the Lagrangian of the system as a function of \( r \), \( \phi \) (the angle around the bowl), and their derivatives. You may need to do a little work to get the gravitational potential energy correct.

(b) Write and simplify the Euler-Lagrange equations.

(c) Under what conditions (if any) will the marble move in a circular “orbit” around the bowl?

4. 7.31
5. 7.49

6. A particle moves in Cartesian coordinates with a potential energy: \( U(z) = mgz \). The Lagrangian will remain invariant under the transformation

\[
\begin{align*}
x & \rightarrow x + \epsilon y \\
y & \rightarrow y - \epsilon x
\end{align*}
\]

(a) Assuming \( \epsilon \) is small (e.g. \( \epsilon^2 \approx 0 \)), compute \( v^2 \) in the transformed frame, and compare to the original frame.

(b) What is the corresponding conserved Noether quantity?