Cosmology – HW 1

HW 1
Due January 18, 2016

Please answer all questions clearly and concisely. You are strongly encouraged to discuss the homework with your classmates, but you must complete the written homework by yourself, and of course, the material you submit must be your own.

1. Ryden 2.2
2. Ryden 4.1
3. Ryden 5.3
4. Consider a universe in which $\Omega_M = 0.32$, but which is otherwise empty.
   (a) In units of the Hubble time ($1/H_0$), how old is the universe now, $t_0$? You will need to do a numerical integration, so please include your code.
   (b) Now adjust your model so that it contains a matter component $\Omega_M = 0.32$, and a “cosmological constant” component, $\Omega_\Lambda = 0.68$ (with $w=-1$). How old is this universe?
   (c) Grad Students only: For each of the above models, plot a (y-axis) versus t (x-axis). Be sure that $a = 1$ and $t =$now represent the same point on the plot for both models, and that the two models are superimposed. Time should be expressed in units of $H_0^{-1}$.
5. The currently accepted value of the Hubble constant is approximately 67 km/s/Mpc. Consider a galaxy observed with a redshift, $z = 0.05$. Note that this is sufficiently small that you may assume $H_0$ to be a constant during the entire duration of the light travel time.
   (a) How far away is the galaxy?
   (b) How long ago was the light emitted from the galaxy?
   (c) Imagine a Balmer-$\alpha$ photon was emitted from this galaxy (If you don’t remember what this is, I strongly recommend you look it up). At what wavelength will you observe it here on earth?
6. Imagine a matter only universe in which $\Omega_M = 0.99999$ at $t = 1s$. While we normally set $t_0$ to “now,” the “now” for this problem should be 1 second after the big bang.
   (a) What was the Hubble constant 1s after the big bang in this universe? Given the value of $\Omega_M$, you may assume the universe behaved as an Einstein-deSitter universe up until that point.
   (b) What was the critical density 1s after the big bang in this universe?
   (c) By what factor will the universe expand from $t = 1s$ to $t = 10Gy$? You will need to do a numerical integration.
   (d) What will $\Omega_M$ be 10 billion years after the big bang?
(over)
7. Suppose you live in an expanding universe with the same physics as our own but (potentially) different cosmological parameters. Through astronomical means, it is established that the universe is between 10 and 11 Gy old.

(a) Suppose the universe contains only matter, and thus there are only two relevant cosmological parameters: $\Omega_M$ and $h$. On a single plot relating the two parameters, illustrate all possible combinations of these two parameters.

This should be done on a computer, not by hand, and ranges should include $0 < \Omega_M < 2$ and $0 < h < 2$.

(b) Grad Students only: Suppose instead, we consider universes that are geometrically flat and have a cosmological constant:

$$\Omega_M + \Omega_\Lambda = 1$$

The universe can still be described by two parameters, $\Omega_M$ and $h$. Do the same analysis in part a, and overplot the ranges of parameters for these universes over the results from part a.